

An alternative competing risk model to the Weibull distribution in lifetime analysis

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Abstract

A simple competing risk distribution as a possible alternative to the Weibull distribution in lifetimes analysis is proposed. This distribution is the minimum between an exponential and a Weibull distributions. First, its main characteristics are presented. Then the estimation of its parameters are considered through maximum likelihood and through Bayesian inference. Decision tests to choose between a Weibull distribution and this competing risk distribution are presented. And this alternative distribution is compared to the Weibull distribution from numerical experiments on both real and simulated data sets.

Keywords: Failure Time Distribution; Aging; Weibull Distribution; Accidental Failure; Competing risk Model; EM algorithm; Bayesian Inference.

1 Introduction

In a reliability context, the most employed lifetime distributions are the exponential distribution and the Weibull distribution (see for instance Meeker and Escobar, 1998). The exponential distribution $\mathcal{E}(\eta)$ whose reliability function is

$$R_E(t) = \exp\left(-\frac{t}{\eta}\right), \quad (1)$$

the scale parameter η being the inverse of the constant hazard rate, is modelling accidental failure times of a no aging material cleared of infant mortality defects. While the versatile Weibull $\mathcal{W}(\eta, \beta)$ distribution, with reliability function

$$R_W(t) = \exp\left[-\left(\frac{t}{\eta}\right)^\beta\right] \quad (2)$$

and hazard rate

$$h_W(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \quad (3)$$

can be used for modelling infant mortality defects when the shape parameter $\beta < 1$ or aging when $\beta > 1$. Note that when $\beta = 1$ the Weibull distribution reduces to an exponential distribution with scale parameter η .

Reliability feedback experience data are often modelled with the Weibull distribution and the important question is to decide if $\beta = 1$ versus $\beta < 1$ when concerned with infant mortality or $\beta = 1$ versus $\beta > 1$ when concerned with aging. This question can be solved using likelihood ratio tests (see d'Agostino and Stephens, 1986). For simplicity in the following, we consider that we are interested to model a possible aging of a material cleared of infant mortality defects. If, for instance, aging is diagnosed, then further statistical inference is made assuming that the observed failure times are arising from a Weibull distribution whose parameters are to be estimated. Acting in such a way, it implies that the occurrence of accidental failures can be regarded as negligible as

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